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Current Capabilities for Simulating the Extreme Distortion of Thin Structures Subjected to Severe Impacts

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SUMMARY

The explicit transient dynamics technology in use today for simulating the impact and subsequent transient dynamic response of a structure has its origins in the "hydrocodes" dating back to the late 1940's. The growth in capability in explicit transient dynamics technology parallels the growth in speed and size of digital computers. Computer software for simulating the explicit transient dynamic response of a structure is characterized by algorithms that use a large number of small steps.

In explicit transient dynamics software there is a significant emphasis on speed and simplicity. The finite element technology used to generate the spatial discretization of a structure is based on a compromise between completeness of the representation for the physical processes modelled and speed in execution. That is, since it is expected in every calculation that the deformation will be finite and the material will be strained beyond the elastic range, the geometry and the associated gradient operators must be reconstructed, as well as complex stress-strain models evaluated at every time step. As a result, finite elements derived for explicit transient dynamics software use the simplest and barest constructions possible for computational efficiency while retaining an essential representation of the physical behavior. The best example of this technology is the four-node bending quadrilateral derived by Belytschko, Lin and Tsay (1984).

Today, the speed, memory capacity and availability of computer hardware allows a number of the previously used algorithms to be "improved." That is, it is possible with today's computing hardware to modify many of the standard algorithms to improve their representation of the physical process at the expense of added complexity and computational effort.

The purpose of this presentation is to review a number of these algorithms and identify the improvements possible. In many instances, both the older, faster version of algorithm and the improved and somewhat slower version of the algorithm are found implemented together in software. Specifically, the following seven algorithmic items are examined:

1) The invariant time derivatives of stress used in material models expressed in rate form.
2) Incremental objectivity and strain used in the numerical integration of the material models.
3) The use of 1-point element integration versus mean quadrature.
4) Shell elements used to represent the behavior of thin structural components.
5) Beam elements based on stress-resultant plasticity versus cross-section integration.
6) The fidelity of elastic-plastic material models in their representation of ductile metals.
7) The use of Courant subcycling to reduce computational effort.

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INVARIANT TIME DERIVATIVE OF STRESS

To account for the fact that bodies subjected to large displacements undergo significant rigid body rotations, material models in rate form rely on an *invariant* time derivative of the stress. For example, a linear elastic material expressed in terms of rates has the form:

$$\begin{aligned} \check{t}^{rs} &= \dot{t}^{rs} - \delta^{rk}\omega_{km}t^{ms} - \delta^{sk}\omega_{km}t^{mr} \\ &= C^{rsmn}d_{mn}\,, \end{aligned} \tag{1}$$

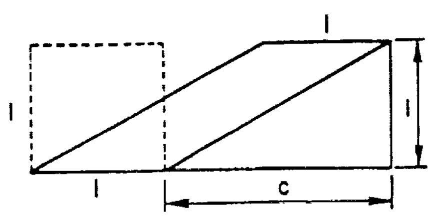
where ω_{km} is a rotation rate, d_{mn} is the stretching and C^{rsmn}, the material's elastic modulus. To date the majority of simulations have used the Jaumann invariant derivative which uses the spin for the rotation rate, $\omega_{km} = (\upsilon_{k,m} - \upsilon_{m,k})/2$.

This formulation is very sensitive to shear strains that are greater than 20% in the presence of rotations (Dienes, 1979).

A more accurate invariant time derivative is the Green-McInnis derivative which is based on the rotation from the polar decomposition of the deformation gradient, $\omega_{km} = \dot{R}_{kn}R^{n}_{\ m}$, where $F_{\alpha}^{\ m} = R^{mn}\ U_{n\alpha}$ (Dienes, 1979).

Computing the uniform shearing of a 1×1 coupon demonstrates the difference in predicted behavior using kinematic hardening plasticity for these two invariant time derivatives (Fig. 1).

Uniform Shear



$$\text{Stretching: } \begin{bmatrix} 0 & D \\ D & 0 \end{bmatrix}$$

$$\begin{aligned} VOL &= 1 \\ \omega &\neq 0 \\ \dot{t}_{12} &= 2\mu d_{12} + \omega_{12}(t_{22} - t_{11}) \\ \dot{t}_{11} &= +2\omega_{12}t_{12} \\ \dot{t}_{22} &= -2\omega_{12}t_{12} \end{aligned}$$

Non-Radial Loading in Plasticity

(Note: $\ln(\ell/\ell_o) = 5 \Rightarrow C = 10$)

Figure 1 - Uniform shearing of a 1×1 coupon

An examination of the shear stress as a function of shear strain produces very interesting results (Fig. 2). The results based on using the Green-McInnis derivative in Fig. 2 are denoted by "Dienes." As can be seen, extreme distortions using the Jaumann derivative lead to physically unrealistic stress variations that change sign. The monotone increasing curve denoted by "Dienes" exhibits a physically realistic stress-strain representation. This anomalous behavior exhibited by the Jaumann derivative must be avoided in a simulation if practical results are to be obtained.

For the sake of reliability, today's software will offer the Green-McInnis form of the invariant time derivative of stress along side of the Jaumann derivative for material models based on rate formulations.

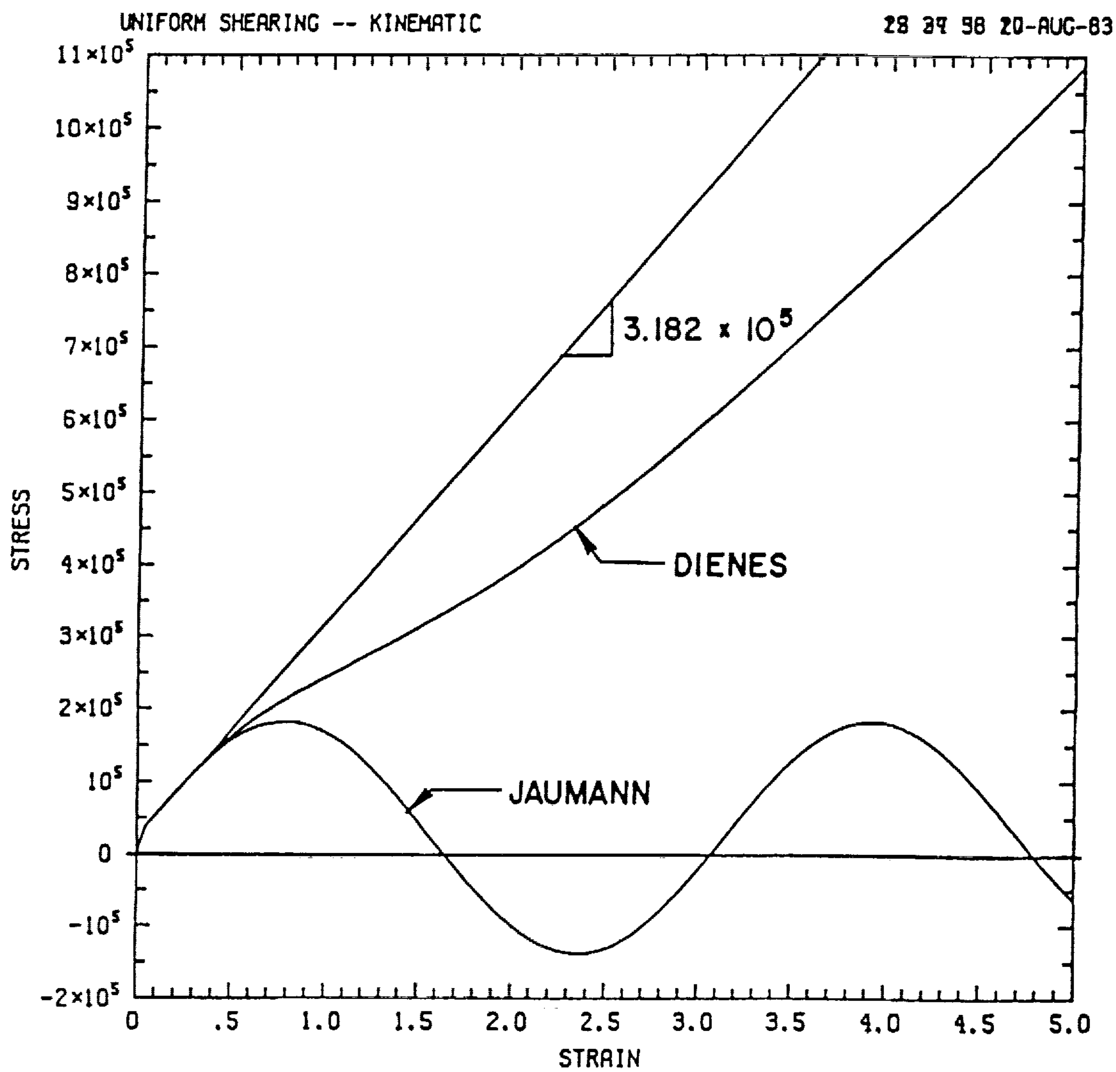


Figure 2 - Kinematic hardening plasticity predictions of shear

INCREMENTAL OBJECTIVITY FOR STRAIN

In numerical simulations differential expressions are replaced with incremental expressions. Incremental expressions for such things as rotation and strain increments can produce errors even for small steps if the properties of the differential expression are lost. For example, when the time derivative of an *orthogonal rotation* is approximated, it is very easy to lose the orthogonal properties associated with the differential form.

The same thing can happen when integrals of strain rates are replaced by sums of strain increments, particularly when finite strains are involved. Traditionally, a strain increment is obtained from the stretching with $\Delta E_{mn} = \Delta t \; d_{mn}$, $d_{mn} = (\upsilon_{m,n} + \upsilon_{n,m})/2$.

This approach to generating strain increments can be sensitive to cyclic strain histories in the presence of rotations.

A more accurate, but computationally more costly approach, is to base the strain increment on the symmetric stretch tensor U obtained from the polar decomposition of the deformation gradient, $F_{\alpha}^{\ m} = R^{mn} \; U_{n\alpha}$. This latter approach has been termed *strong objectivity* by M. Rashid (1992), who refers to the traditional approach as *weak objectivity.*

Computing the hoop vibration of a rotating ring provides a good example of the contrasting results that can come from the use of the faster, classical formulations that are weakly objective and the more accurate and costly formulations that are strongly objective. Figure 3 shows an elastic ring given an initial angular velocity of 4,000 radians per second.

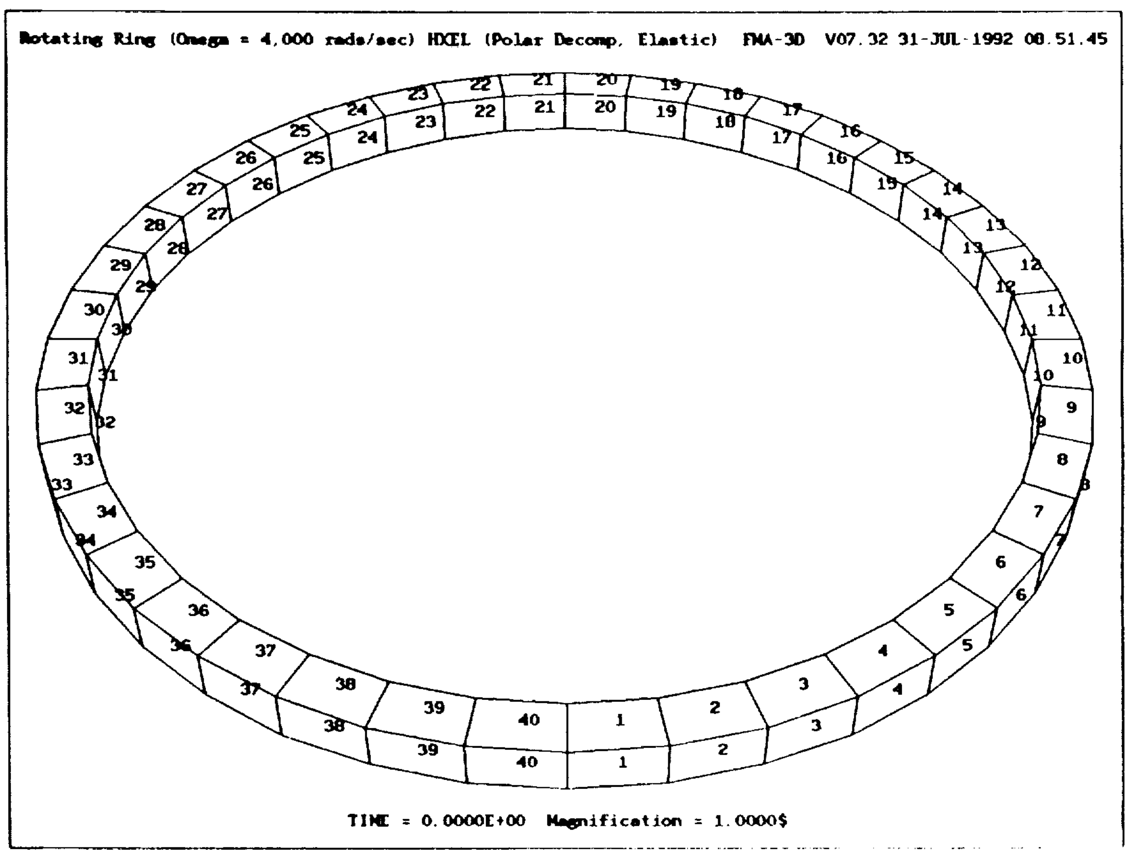


Figure 3 - An elastic ring with an initial 4,000 rad/s angular velocity

The ring rotates approximately 360 degrees in 1.5 milliseconds. During that time the ring oscillates radially five times (Fig. 4). Of particular interest is the increase in kinetic energy with time (Fig. 4). This is a closed system to which no additional energy is being added. The increase in kinetic energy reflects the accumulation of errors from the strain increments in the classical formulation that is weakly objective in spite of the very small steps in the algorithm.

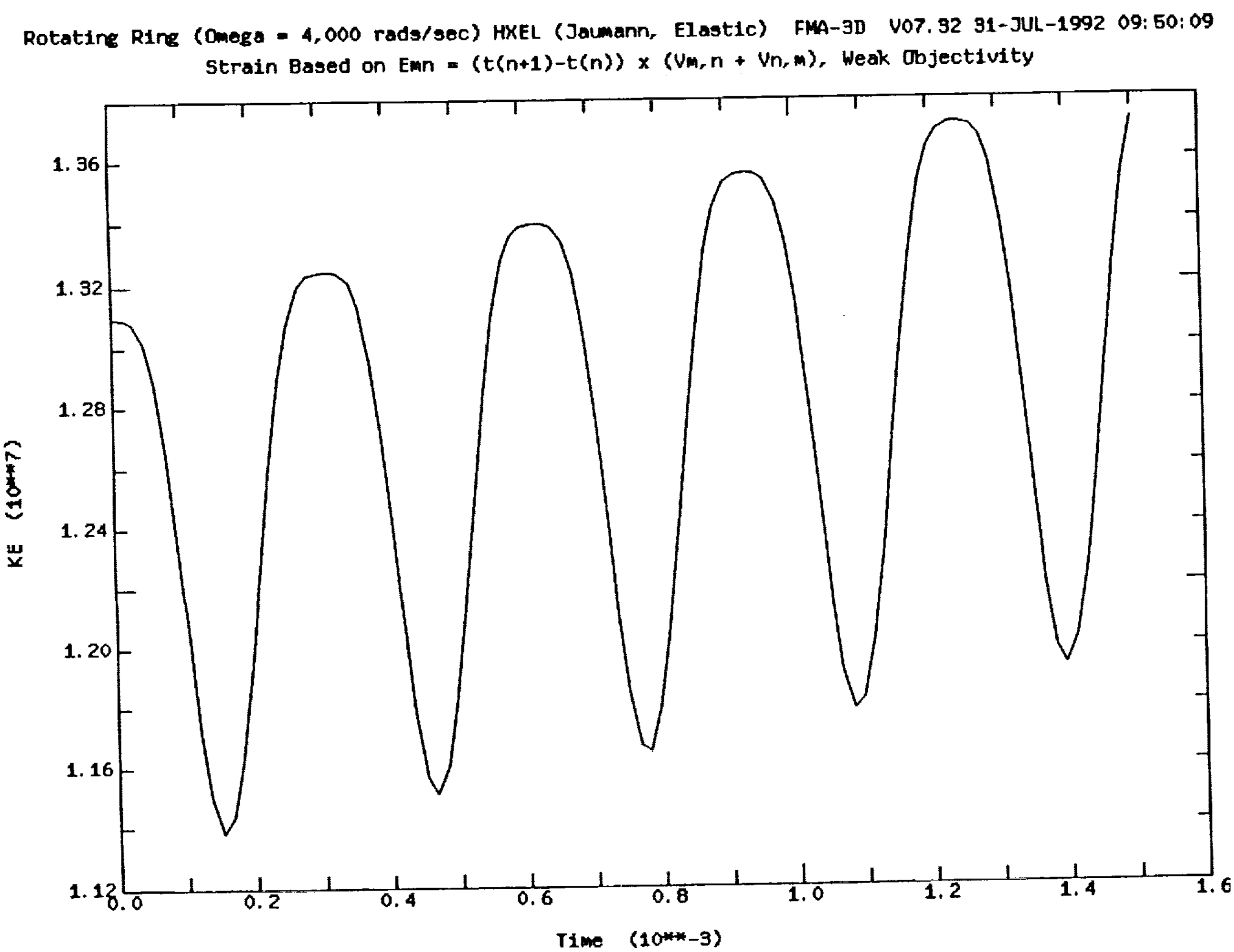


Figure 4 - Kinetic energy response with only weak objectivity

The same computation using a formulation that is strongly objective in the sense of Rashid shows the expected response (Fig. 5); namely, a kinetic energy that oscillates between two limits that are constant in time. This latter calculation shows an exemplary energy exchange between kinetic energy and internal strain energy as the ring rotates and oscillates radially.

In spite of the increased expense, today's software should offer as the default a strain increment formulation meeting the requirements for strong objectivity.

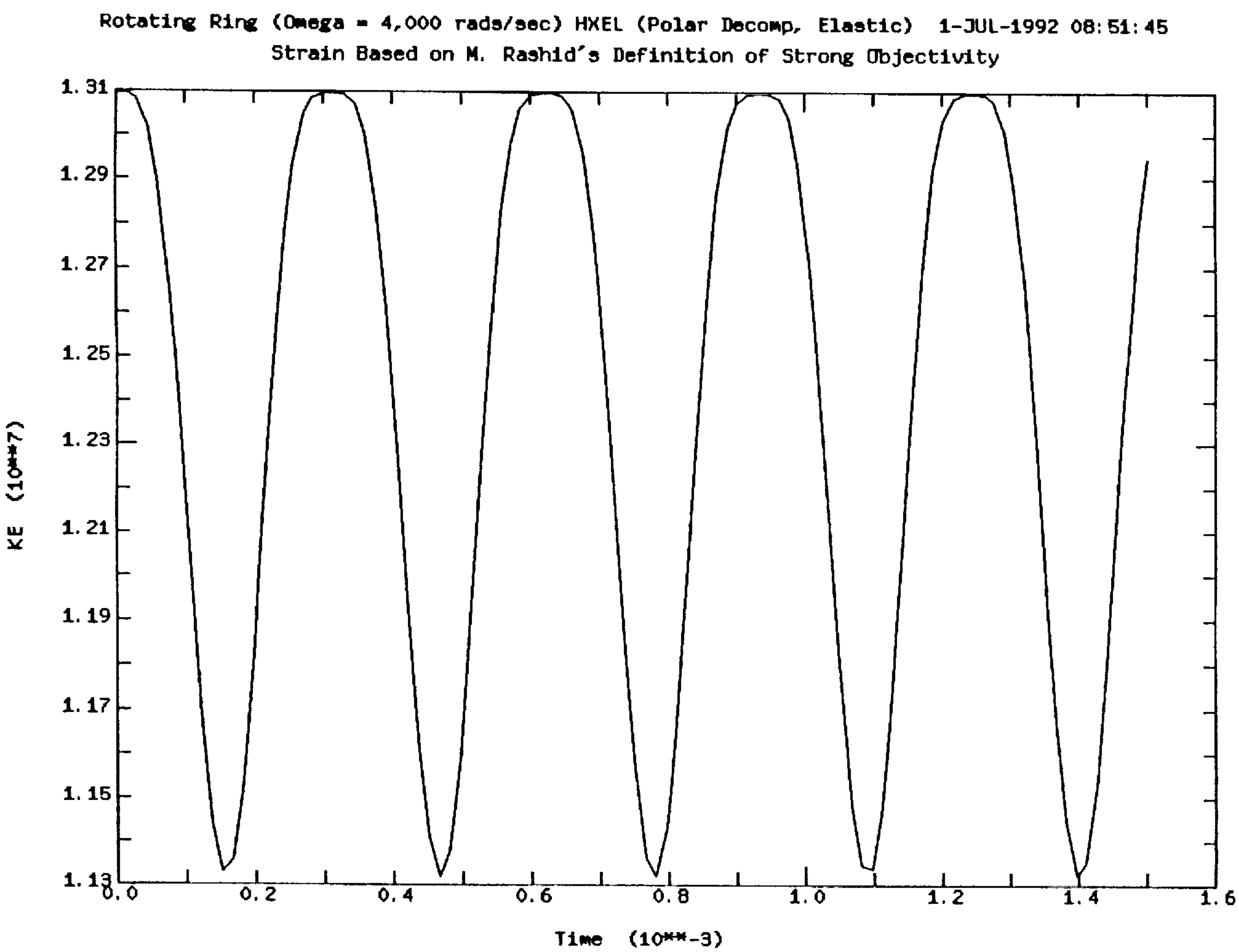


Figure 5 - Kinetic energy response with strong objectivity

ELEMENT INTEGRATION

In the interest of speed, explicit transient dynamic computer programs routinely use "constant stress" elements. In effect, only the uniform strain states of the element are used to represent the structures behavior. Strain states and, consequently, stress states that vary over the element are ignored. Such approximations result in complex, short wavelength, stress-free deformation patterns that are called *hourglass modes*.

The quickest way to obtain a constant stress element is to evaluate the integrals over the area or volume of the element defining the gradient and divergence operators with a single integration point; hence, the terminology *1-point integrated* elements. Unfortunately, such elements can fail the Iron's patch test. That is, a collection of irregularly-shaped elements subjected to a linear motion on the boundary will not produce a constant state of strain in the interior and, therefore, the collection of irregularly-shaped elements will not produce a constant state of stress.

A more accurate approach is to evaluate the integrals over the area or volume of the element defining the gradient and divergence operators exactly for a constant state of stress, effectively a projection calculation. The result is a much more reliable and well-behaved simulation, albeit requiring the execution of a greater number of algebra expressions. The *mean quadrature* elements while still constant strain elements, pass the Iron's patch test.

The greatly reduced excitation of the hourglass modes and the greatly improved hourglass control offered by the mean quadrature integration over the 1-point integration virtually renders obsolete the older 1-point integration technology. (In the special case of two-dimensional continuum elements, both approaches yield the same results.)

Current users of explicit transient dynamic software should only expect to use 1-point integrated elements in place of elements based on mean quadrature to obtain "quick and dirty" results.

In explicit transient dynamics software there is a significant emphasis on speed and simplicity. That is, since it is expected in every calculation that the deformation will be finite and the material will be strained beyond the elastic range, the geometry and the associated gradient operators must be reconstructed, as well as, complex stress-strain models evaluated at every time step. As a result, finite elements derived for explicit transient dynamics software use the simplest and barest constructions possible for computational efficiency while retaining an essential representation of the physical behavior. The best example of this technology is the four-node bending quadrilateral derived by Belytschko, Lin and Tsay (1984).

The BLT element is based on a constant stress assumption coupled to a flat plate geometric approximation of what is usually a warped geometry (the element's geometry is warped when the four nodal points do not define a flat surface). In certain situations the BLT element exhibits shortcomings in representing the response of a structure. Two examples are the bending of a twisted beam (Example 1), and the response of a hemisphere loaded by opposing forces across the hemisphere's diameter (Example 2).

A four-node bending quadrilateral has been developed that exhibits the expected behavior in these two examples. The element has two properties that provide the expected response:

1) a warping deformation that possesses proper structural stiffness as opposed to being an hourglass mode, and

2) a derivation that is based on the actual geometry of the element as opposed to treating the geometry as flat.

Example 1. The tip-loaded, twisted cantilever beam is an example of a structure with a nonplanar geometry that can be nontrivial to reproduce with the finite element technology available today in explicit transient dynamic software (Fig. 6).

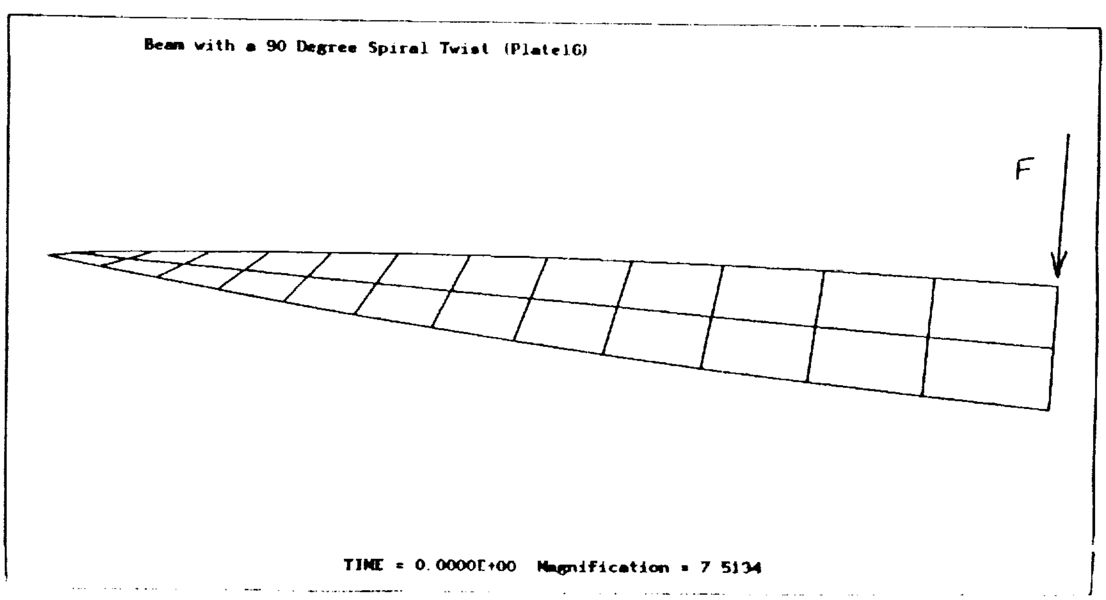


Figure 6 - Twisted beam with a constant tip load

The tip load is applied suddenly at time equal to zero and held constant in magnitude and orientation. Based on a length of 12.0 in., a width of 1.1 in., a thickness of 0.32 in. and the selected properties (a Young's modulus of 2.9×10^7 psi, a Poisson's ratio of 0.22, and a density of 2.5×10^{-4} lbf-sec^2/in^4), the static deflection equals 0.005424 in. (MacNeal and Harder, 1985). The fundamental period equals 8.0 milliseconds.

As can be seen from Fig. 7, the BLT element with zero hourglass stiffness predicts an unacceptably large deflection amplitude and response period; see also the results reported by Belytschko, Wong and Chiang (1989).

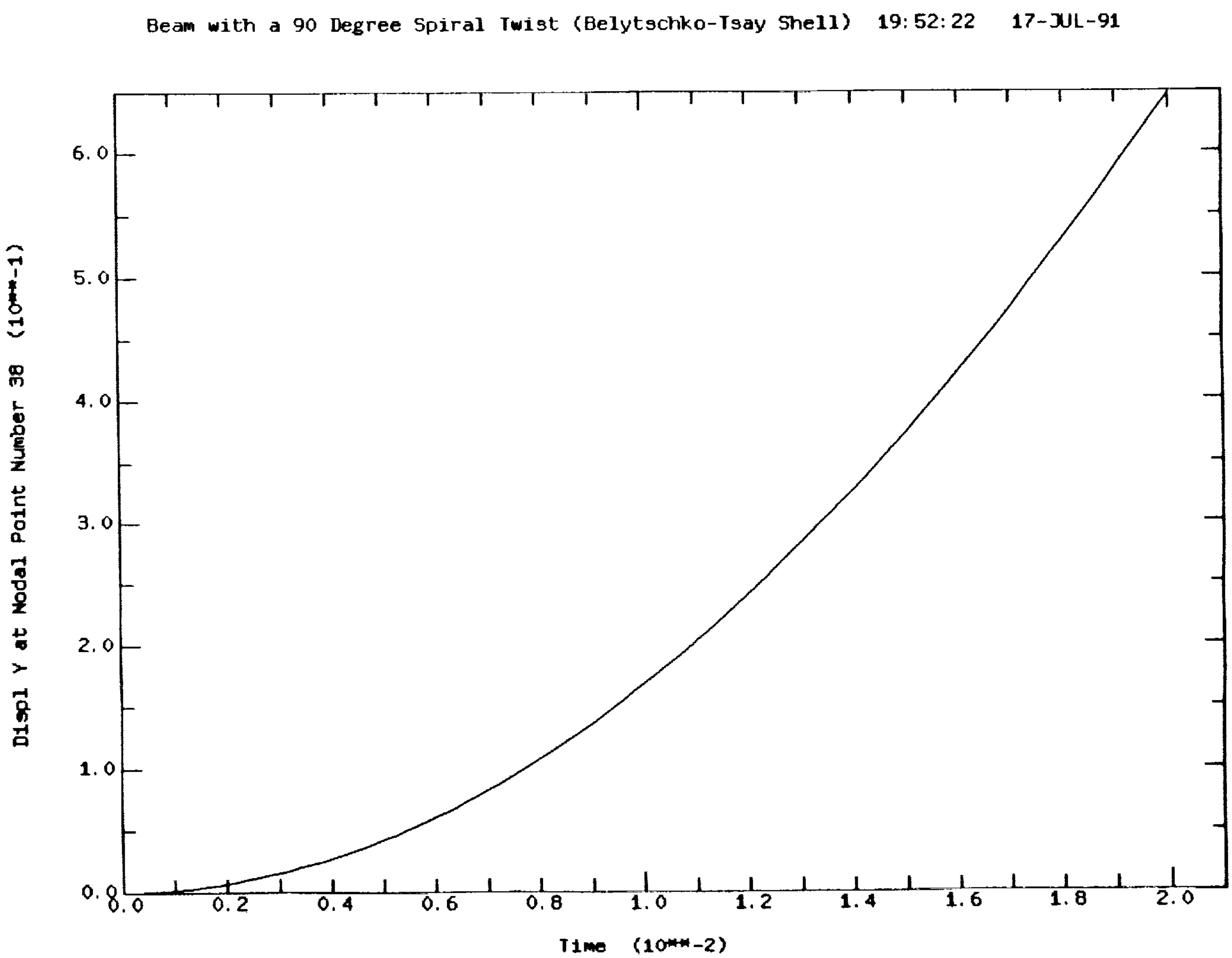


Figure 7 - Beam tip motion prediction using Belytschko-Tsay shell

The technology developed by KEY Associates predicts very nearly the exact amplitude and period (Fig. 8).

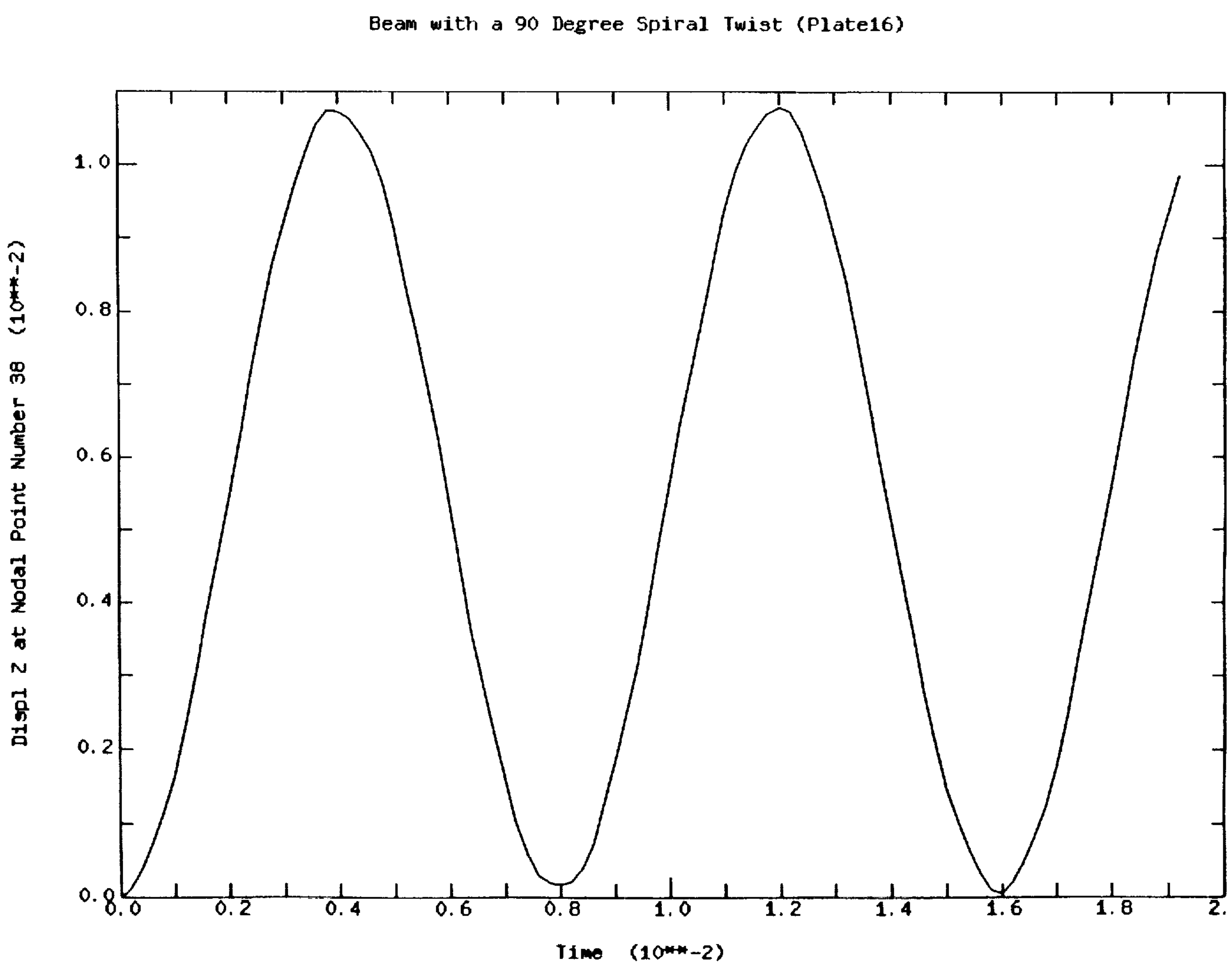


Figure 8 - Beam tip motion prediction using improved four-node shell

Example 2. A hemisphere loaded by two sets of diametrically opposed forces in the plane of the equator, separated by 90 degrees and alternating in sign, is a problem in which both bending and membrane deformation occur. The loads enter the structure by generating moments including warping or twisting moments in the elements adjacent to the loads.

The loads are applied suddenly at time equal to zero and held constant in magnitude and orientation (Fig. 9).

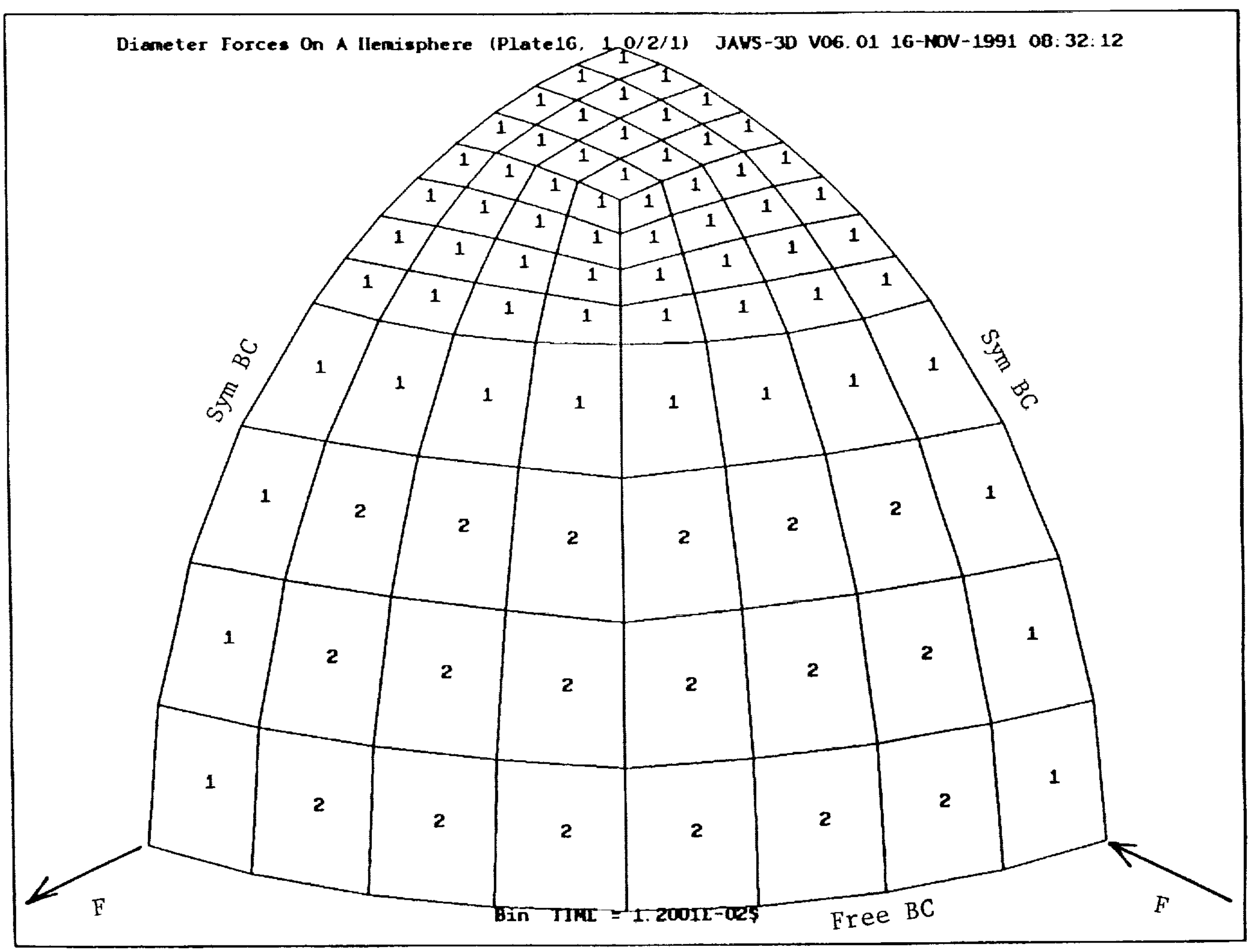


Figure 9 - Quadrant of a hemisphere loaded across two diameters

Based on the hemisphere's radius of 10.0 in., a thickness of 0.04 in. and the selected properties (a Young's modulus of 6.825×10^7 psi, a Poisson's ratio of 0.3, and a density of 2.5×10^{-4} 1bf-sec^2/in^4), the static deflection equals 0.094 inches (MacNeal and Harder, 1985).

As can be seen from Fig. 10, the BLT element with zero hourglass stiffness predicts an unacceptable cyclic accumulation of displacement; see also the results reported by Belytschko, Wong and Chiang (1989).

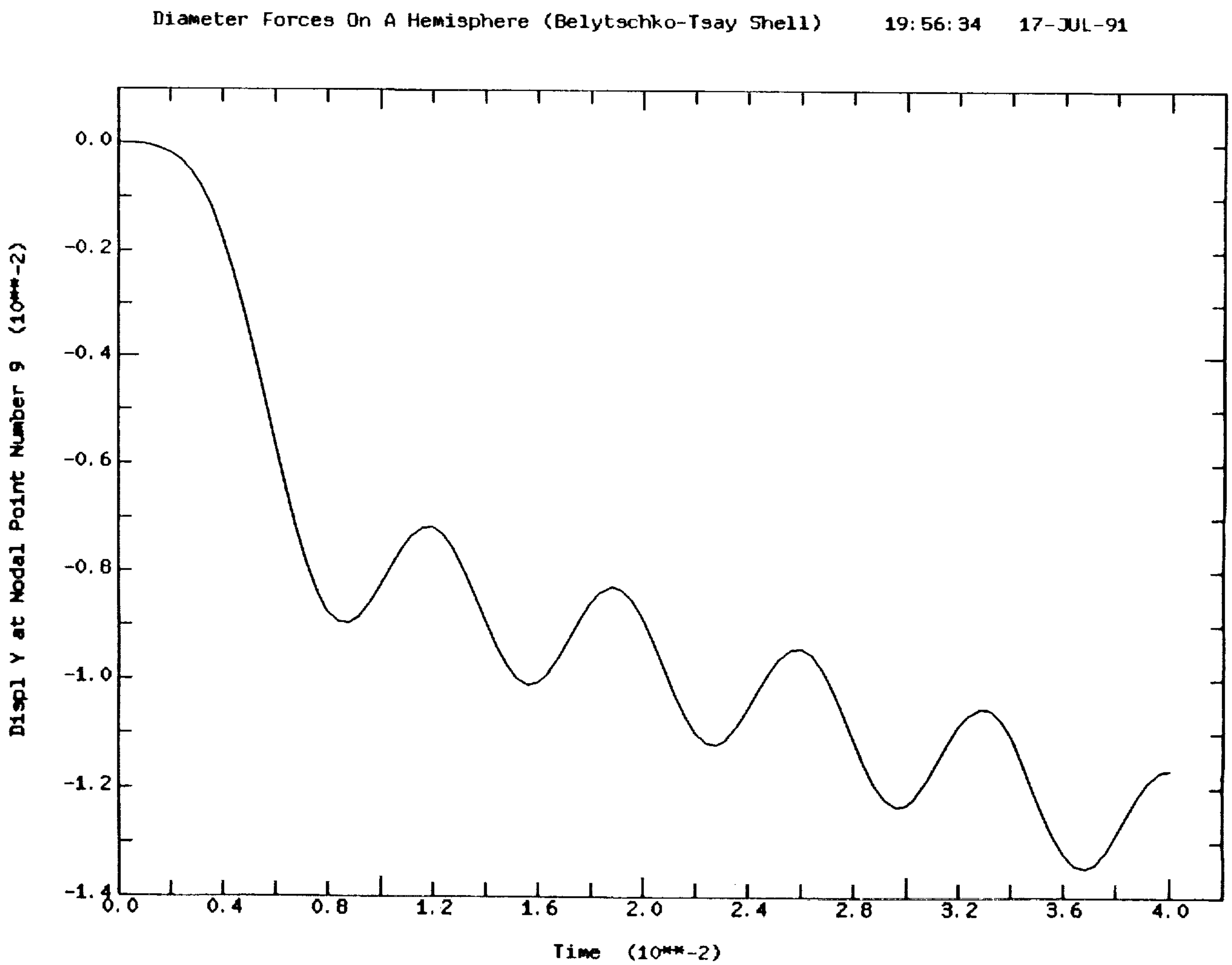


Figure 10 - Radial motion prediction using Belytschko-Tsay shell

The technology developed by KEY Associates predicts a steady periodic (constant amplitude) result that is within 0.2% (Maximum_Internal_Energy / 4.0) of the expected deflection (0.094 inches) (Fig. 11).

Remarks. This element is suitable for both large in-plane and bending strains. These particular calculations were carried out without any hourglass control suggesting this element will not be overly sensitive to the development of hourglass distortions. In addition, this technology is based on six (6) degrees of freedom per nodal point meaning that no further numerical constraints or adaptations are needed to eliminate the "drilling" rotation to obtain satisfactory results. Modeling such additional physical features as edge beams, or modeling folded plate structures does not present any particular difficulty.

The low level of loading in both of these examples results effectively in infinitesimal strains. The material response is linear elastic. However, both of these examples require a proper computation of the gradient and divergence operators to obtain the correct results. The examples are sensitive indicators of the correctness of the representation for the element geometry and the element twisting stiffness.

While the BLT four-node bending quadrilateral remains a computationally efficient element, there are numerous applications for which an accurate representation of the warped geometry and the twisting stiffness is essential to obtaining a satisfactory result. Without a doubt, up-to-date software should contain an efficient four-node bending quadrilateral with the capabilities discussed here.

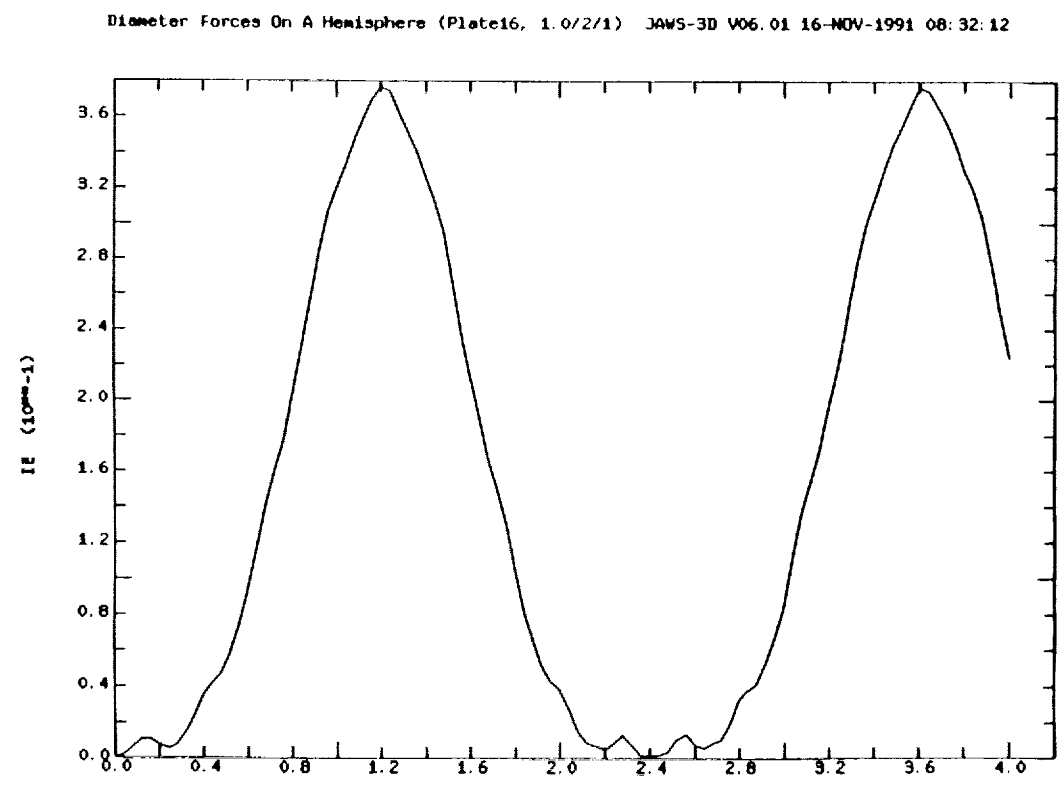


Figure 11 - Radial motion prediction (= I E/4) using improved four-node shell

BEAM ELEMENTS

Beam elements, while on the surface appearing to be very elementary structural components, can be very intense from a computational standpoint when finite strains and inelastic material behavior are modeled. A portion of the computational complexity comes from the need to evaluate the stress-strain behavior at many points over the cross section (16 integration stations is a typical number). Membrane, bending and torsional stress resultants are produced from weighted sums over these points.

A very fast and direct method of obtaining stress resultants for elastic-plastic material behavior is to introduce stress-resultant plasticity. In effect, the beam cross-section is either completely elastic or completely plastic. For simulations in which the beam is deformed well into the plastic range, considerable computational effort can be eliminated with stress-resultant plasticity (the evaluation of elastic-plastic stress-strain models at numerous individual points over the cross section is not required).

If only a portion of the beam cross section yields plastically, stress-resultants plasticity will not capture any of the detail.

Quality software seeking to provide options for both rapid results and accurate results will offer both beam formulations.

COURANT SUBCYCLING

The central difference time integrator is only *conditionally* stable. That is, the integration procedure must be used with a time step that is less than the *critical time step*. The critical time step is very nearly equal to the minimum transient time for a disturbance to cross between any two nodal points in the mesh. The algorithm is very effective for an excitation that uses the maximum resolution the mesh can provide; for example, a shock wave.

There are two very common circumstances when an explicit time integration algorithm becomes less efficient: first, when there are significant differences in the spacing between nodal points over the mesh and, second, when there are significant differences in material stiffness over the mesh. It is not uncommon to have differences in critical time steps as much as a hundred to one over a mesh. The consequence is that the central difference time integration must function with the smallest critical time step.

Fortunately, Courant subcycling may be introduced in order to recover much of the efficiency offered by explicit algorithms. In Courant subcycling each finite element is integrated with the largest time step permitted by the local critical time step. Thus, small, stiff elements are integrated with many small time steps, and elsewhere in the mesh large, soft elements are integrated periodically with their larger time steps.

The benefit is significant since the time to perform a calculation drops and the answers are more accurate since the central difference algorithm performs as close to the critical step as is practicable everywhere. Figure 12 shows an example of a domain in which elements are integrated with two different time steps. In this case the ratio is only 2 to 1, but the principle is demonstrated.

Clearly, software intended for a wide variety of applications should provide Courant subcycling.

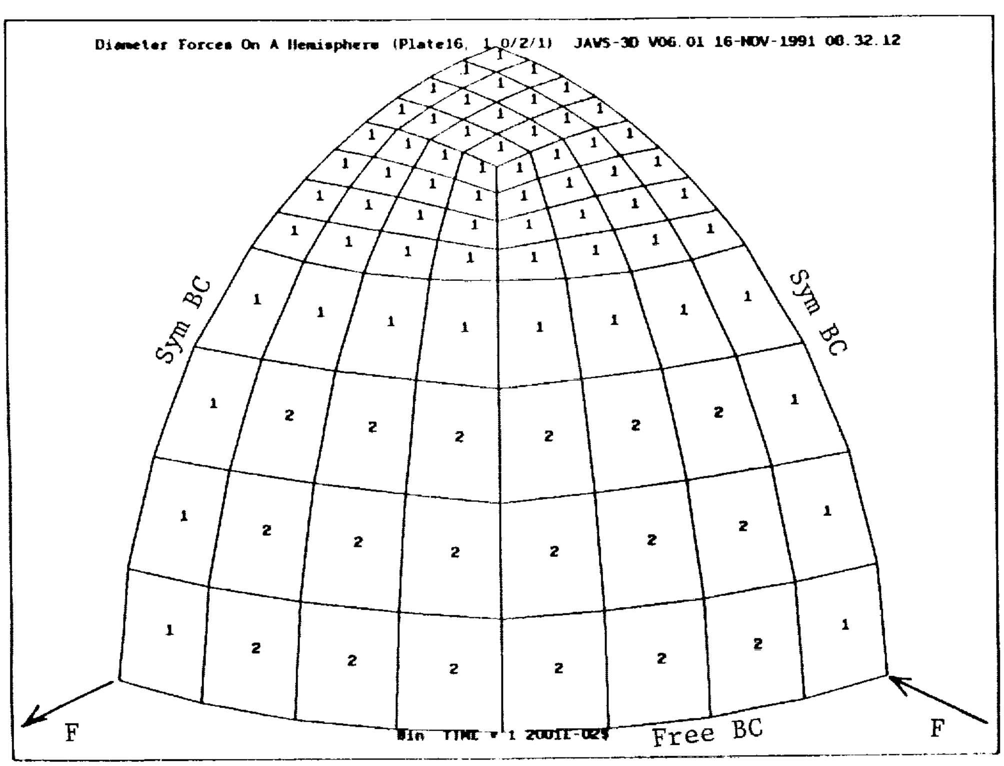


Figure 12 - Element integration bins for Courant subcycling

PLASTICITY IMPLEMENTATIONS

Traditional elastic-plastic material models are based on a linear representation of plastic hardening and radial-return numerical integration. While an exceptionally effective approach for large plastic strains, it is somewhat approximate for moderate plastic strains (plastic strains less than a ten times elastic strains. Ductile metals can exhibit a very "rounded" stress-strain curve when first yielding plastically. An elastic-linear strain hardening model with its transition from elastic to plastic response at a sharp corner does not mimic a gradual transition to large plastic strains.

One should expect a variety of material models in a crash simulation program including the basic linear strain hardening model and a model for plastic yielding that has a smooth transition from elastic to plastic response. For small plastic strains the models available should include sub-stepping where the time step within the material model is broken into smaller steps to control the numerical integration error.

SOFTWARE SURVEY

The following two tables (Tables 1 and 2) are a modest survey of the crash simulation software represented by the speakers at this NASA Workshop on Crashworthiness Technology. In brief, the software surveyed is as follows:

1. FMA-3D is a full-feature program available commercially from KEY Associates, 1851 Tramway Terrace Loop NE, Albuquerque, NM 87122 that include subcycling.

2. DYNA-3D is a full-featured program available through a technology sharing agreement from the Lawrence Livermore National Laboratory, Attn: Robert Whirley, P.O. Box 808, Livermore, CA 94550.

3. PRONTO-3D is a limited-feature program available with a license agreement from Sandia National Laboratories, Attn: Steven Attaway, Organization 1425, Albuquerque, NM 87185.

4. SUPERWHAMS is a full-featured program soon to be available commercially from KBS2, Suite 112, 455 South Frontage Road, Burr Ridge, IL 60521.

5. DYCAST is a limited-feature program based on an implicit integration algorithm available commercially from Grumman Aerospace Corporation, Attn: Allan Pifko, MS A08-35, Bethpage, NY 11714.

Table 1. Technology Availability

	FMA-3D	DYNA	PRONTO
Time Derivative			
Jaumann	Yes	Yes	No
Green-McInnis	Yes	No	Yes
Strain Increment			
Weak Objectivity	Yes	Yes	No
Strong Objectivity	Yes	No	Yes
Element Integration			
1-Point	No	Yes	No
Mean Quadrature	Yes	Yes	Yes
4-Node Shells			
BLT Quad	Yes	Yes	Yes
Improved Quad	KEY	EWQ	No
Beam Elements			
Illyshin Plasticity	(i/i)	Yes	No
Integrated	Yes	Yes	No
Plasticity			
Linear Hardening w/ Radial Return	Yes	Yes	Yes
Smooth Hardening w/ Sub-Stepping	Yes	"Yes"	No
Time Subcycling	Yes	No	No

Table 2. Technology Availability

	SUPER WHAMS	DYCAST
Time Derivative		
Jaumann	Yes	(no solid
Green-McInnis	No	elements)
Strain Increment		
Weak Objectivity	Yes	(no solid
Strong Objectivity	No	elements)
Element Integration		
1-Point	No	(no solid
Mean Quadrature	Yes	elements)
4-Node Shells		
BLT Quad	Yes	(Δ l/d&r)
Improved Quad	BWCQ	(inf strain)
Beam Elements		
Illyshin Plasticity	No	No
Integrated	Yes	Yes
Plasticity		
Linear Hardening w/ Radial Return	Yes	No
Smooth Hardening w/ Sub-Stepping	No	Yes
Time Subcycling	Yes	Implicit

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